Report 2151

AD632077

DEPARTMENT OF THE NAVY

HYDROMECHANICS

AERODYNAMICS

STRUCTURAL MECHANICS

APPLIED MATHEMATICS

ACOUSTICS AND VIBRATION

PITCHING MOTIONS OF A MOORED SUBMERGED MINE IN WAVES

by

J. N. Newman
P. Plaia
E. E. Zarnick

Distribution of this document is unlimited.

HYDROMECHANICS LABORATORY
RESEARCH AND DEVELOPMENT REPORT

March 1966

Report 2151

DAVID TAYLOR MODEL BASIN
WASHINGTON, D. C. 20007

PITCHING MOTIONS OF A MOORED SUBMERGED MINE IN WAVES

by

J. N. Newman

P. Plaia

E. E. Zarnick

March 1966

Report 2151
NOL WR 5-0044

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

NOTATION

A	Incident wave amplitude (single-amplitude)
c	Mooring cable length
g	Gravitational acceleration
h	Depth of submergence of body axis
I	Pitch moment of inertia of body
I	Pitch added moment of inertia
K	Wave number, $K = \omega^2/g = 2\pi/\lambda$
ℓ	Length of mooring arm, measured vertically from body axis
M	Body mass
P	Probability function
p	Fluid pressure
$S(\xi)$	Sectional area of body
$S(\omega)$	Wave spectrum
T	Mooring cable tension
t	Time
V_n	Normal velocity on body surface
$\mathcal{V}$	Volume of body
x,y,z	Cartesian coordinates fixed in space
β	Incident wave heading angle ($\beta = 0$ for head waves)
γ	Angle of mooring cable from vertical
ζ	Vertical Cartesian coordinate fixed in body
ζ_o	Heave displacement
ζ_{CG}	Vertical coordinate of body center of gravity
η	Transverse Cartesian coordinate fixed in body
θ	Pitch angle
λ	Wavelength
ξ	Longitudinal coordinate fixed in body

ξ_o Surge displacement

ξ_{CB} Longitudinal coordinate of center of buoyancy

ξ_{CG} Longitudinal coordinate of center of gravity

ρ Fluid density

σ_θ Root-mean-square pitch amplitude

ϕ Velocity potential of fluid

ϕ_b Velocity potential due to pressure of body

ϕ_i Velocity potential of incident wave

ω Frequency

ω_o Characteristic frequency based on wind speed at 19.5 meters

ABSTRACT

This report describes a theoretical and experimental investigation of the pitching motions of a moored, submerged mine. The theoretical predictions are based upon linearized-wave theory as well as the assumptions that the body is slender and axisymmetric and is ballasted to be at equilibrium in the horizontal plane. The mooring cable is assumed to be massless and inelastic; the fluid is assumed to be inviscid. The theory results in an equation of undamped motion. Parallel experimental results were obtained on a 2-foot long model in wavelengths ranging from 15 to 55 feet, and these results confirm the theoretical predictions except in the vicinity of resonance, where viscous damping is important. Full-scale predictions are made for the root-mean-square pitching motions in Sea States 4 through 7 for two proposed mine configurations at various depths of submergence. The predicted values are from 1 to 9 degrees in Sea State 4, depending on depth and mine configuration, increasing to greater than 25 degrees in Sea State 7.

ADMINISTRATIVE INFORMATION

This work was requested by U.S. Naval Ordnance Lab ltr JM:JR:ich/3900 Ser 6343 to DATMOBAS dated 8 Sep 1964.

INTRODUCTION

In response to a request from the Naval Ordnance Laboratory, an investigation was undertaken to predict the pitching motion in waves of moored mines. The mines are elongated bodies of revolution, about 8 feet long, which are moored with an anchor cable from the nose so as to be in equilibrium at depths of from 50 to 200 feet below the free surface when the axis is horizontal.

Experimental modeling of the problem is complicated because laboratory wavelengths are limited to a maximum of approximately 50 feet, thus implying a scale ratio of at least 1-to-10 between the model and full scale; but a model length of less than 1 foot is impractical, especially when a pitch-measuring gyroscope is incorporated inside the model. In view of this situation it was decided to adopt a scale ratio of 1-to-4, test in the resulting short-wavelength domain, and use a slender-body theory for the pitch motion to extrapolate to longer wavelengths.

Thus a combined theoretical and experimental study is presented with a comparison of the two. The comparison is quite good for the original mine configuration at the short wavelengths (40 feet) where the experiments are reliable. At the longer wavelength (55 feet) there is more scatter of the experimental data, and comparison with the theory is less conclusive. However, the principal deficiency of the theory is its inability to account for the (predominantly viscous) damping near resonance, which occurs for the original mine configuration at much longer wavelengths. To illustrate this, the model was altered physically by increasing the length of the mooring arm, bringing the resonance frequency into the range of experimental wavelengths. The resulting data near resonance is substantially different from the theoretical prediction, so that motions at all wavelengths cannot be accounted for with the theory.

To provide statistical predictions in a realistic seaway the resonance response is determined empirically, using as a rough guide the experimental data with the long mooring arm. The Pierson-Moskowitz[1] spectrum for fully-developed seas is then applied for Sea States 4 through 7 and yields predictions of the root-mean-square pitch amplitude for two mines at various depths of submergence. It is significant that, due to different values of the excess buoyancy of the two mines, the resonant frequencies correspond to very different wavelengths (450 and 1340 feet); thus the relative superiority of the two mines depends critically on the sea state and also on the depth of submergence. It is clear from this comparison that small changes in the volume or weight of such a body can have large effects on the motions.

ANALYTICAL DERIVATION

In this section we shall derive the equations of motion of a moored, submerged body in regular waves. The following assumptions are made:

1. The fluid is inviscid and incompressible.

---

[1]References are listed on page 22.

2. The wave height is small so that all wave and body motions can be linearized.
3. The body is a rigid slender body of revolution.
4. The mooring cable is inelastic; its dynamics can be neglected, and it is attached to the body through a frictionless joint below the body axis.
5. The body is ballasted to lie in the horizontal plane.
6. The fluid is deep.

We utilize two Cartesian coordinate systems, (x, y, z) and (ξ, η, ζ), with (x, y, z) fixed in space and (ξ, η, ζ) fixed in the body (Figure 1). The ξ-axis is the body axis, and when the body is in equilibrium this coincides with the horizontal x-axis. The z-axis is chosen to be vertical and positive upwards. The body is fastened to the mooring cable at the point $(0, 0, -\ell)$, where ℓ is the length of the arm. The centers of buoyancy and gravity of the body are assumed to lie at $(\xi_{CB}, 0, 0)$ and $(\xi_{CG}, 0, \zeta_{CG})$, respectively. (Note that $\zeta_{CB} = 0$ is a result of neglecting the small buoyancy of the arm.) Since the body is ballasted to rest in the horizontal position, it follows that

$$M \xi_{CG} = \rho \forall \xi_{CB} \tag{1}$$

where M is the body mass; ρ is the fluid density; and $\forall$ is the body volume. The tension in the mooring cable is

$$T = \rho g \forall - Mg \tag{2}$$

where g is the gravitational constant.

Restricting ourselves to analysis of motions in the vertical plane, these will consist of a surge displacement ξ_o, heave displacement ζ_o, and pitch displacement θ. However, the restraint of the mooring cable (with the origin directly above the mooring-attachment point) will restrain the heave displacement to a second-order amplitude, which may be neglected in linearized analysis. As a consequence of the surge and pitch displacements ξ_o and θ, the two coordinate systems may be related as follows:

$$x - \xi_o = \xi \cos \theta + \zeta \sin \theta \cong \xi + \zeta\theta \tag{3}$$

$$z = \zeta \cos\theta - \xi \sin\theta \cong \zeta - \xi\theta \qquad [4]$$

Because of the motions ξ_o and θ the mooring cable will be inclined at a small angle γ from the vertical. Assuming the cable length is c, the cable is straight; and the lower end of the cable is at a fixed point (i.e. the anchor). Then

$$c\gamma + \ell\theta \cong \xi_o \qquad [5]$$

$$\gamma = \xi_o/c - \ell\theta/c \qquad [6]$$

We can now write the equations of motion, for the longitudinal force and pitch moment, as follows:

$$- F_x = M \ddot{\xi}_o + \iint p \cos(n,x)\, dS + T\,(\xi_o/c - \ell\theta/c) = 0 \qquad [7]$$

$$- M_\eta = I \ddot{\theta} + \iint p\, [\zeta \cos(n,\xi) - \xi \cos(n,\zeta)]\, dS$$

$$+ T\ell\, [(1 + \ell/c)\theta - \xi_o/c]$$

$$- Mg\, (\xi_{CG} + \zeta_{CG}\, \theta) = 0 \qquad [8]$$

Here dots denote time-derivatives, p is the fluid pressure, cos(n,x) and cos(n,z) are the direction cosines of the (outward) normal to the body surface, and the surface integrals are over that surface, representing the negative of the pressure force and moment exerted by the fluid on the body. The last term in Equation [7], equal to T γ, is the horizontal component of the force exerted on the body by the cable. In Equation [8], I is the body-pitch moment of inertia about the origin; the term involving T is the pitch moment due to the cable tension T and the moment arm ℓ; and the last term is the moment due to the weight of the body acting vertically downward.

To proceed further we must know the pressure p, and it is here that the slender-body assumption becomes necessary. From the Bernoulli equation

$$p = - \rho\, [\frac{\partial\phi}{\partial t} + \frac{1}{2} |\nabla\phi|^2 + gz] \qquad [9]$$

plus a constant, where $\phi(x,y,z,t)$ is the velocity potential, whose gradient is the fluid velocity vector. This potential must satisfy the Laplace equation

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 0 \qquad [10]$$

throughout the fluid; the linearized free surface of the boundary condition

$$\frac{\partial^2\phi}{\partial t^2} + g\,\frac{\partial\phi}{\partial z} = 0 \quad \text{on } z = h \qquad [11]$$

where $z = h$ is the plane of the free surface, and the kinematic boundary condition

$$\frac{\partial\phi}{\partial n} = V_n \qquad [12]$$

on the body, where V_n is the normal velocity of the body surface. For a slender body of revolution, the surge contribution to the normal velocity will be small, compared to the pitch contribution, and

$$V_n \cong -\,\theta\, x \cos(n,z) \qquad [13]$$

The potential ϕ will consist of an incident wave potential ϕ_i; and a body potential ϕ_b, due to the presence of the body. For plane progressive waves of circular frequency ω,

$$\phi_i = \frac{g\,A}{\omega}\exp\left\{K(z-h) + i\,K\,x\cos\beta + i\,Ky\sin\beta - i\omega t\right\} \qquad [14]$$

where the real part is understood. Here A is the wave amplitude; $K = \omega^2/g$ is the wave number; and β is the angle of incidence ($\beta = 0$ for head waves).

Combining Equations [12] to [14], the body potential must satisfy

$$\frac{\partial\phi_b}{\partial n} = -\,\dot{\theta}\, x\cos(n,z) - \frac{\partial\phi_i}{\partial n} \qquad [15]$$

Assuming the body is slender, the body potential for points near the body is then equal to

$$\phi_b = \dot{\theta}\ \xi\ \zeta + \omega\ A\ \zeta \exp\left\{-\ Kh + i\ K\ \xi \cos\ \beta - i\omega t\right\} \qquad [16]$$

plus an odd function of η. Thus, adding Equation [14], the total potential on the body is

$$\phi = \theta\ \xi\ \zeta + (gA/\omega + 2\ \omega\ A\ \zeta)\ \exp\left\{-\ Kh + i\ K\ \xi \cos\ \beta - i\omega t\right\} + O(\zeta^2,\ \eta^2) \qquad [17]$$

plus an odd function of η. Here we have expanded Equation [14] in a Taylor series for small values of the body transverse dimensions. Substituting Equation [17] in the Bernoulli Equation [9], we find that pressure on the body is

$$p = -\ \rho\ \ddot{\theta}\ \xi\ \zeta + (i\rho g\ A + 2i\omega^2\rho\ A\zeta)\ \exp\left\{-Kh + i\ K\ \xi \cos\ \beta - i\omega t\right\} - \rho g\ \zeta + \rho g\ \xi\ \theta \qquad [18]$$

where in Equation [18] and hereafter we consistently delete the nonlinear second order terms in ξ_o, θ, and the wave amplitude A.

The surface integral in Equation [17] can be evaluated as follows from the divergence theorem:

$$\iint p\ \cos(n,x)\ dS = \iiint \frac{\partial p}{\partial x}\ dV = \iiint \left(\frac{\partial p}{\partial \xi} + \theta\ \frac{\partial p}{\partial \zeta}\right) dV$$

$$= \iiint [-\ p\ \ddot{\theta}\ \zeta - \rho g\ K\ A \cos\ \beta\ (1 + 2K\zeta)\ \exp\ (-\ Kh + i\ K\xi \cos\ \beta - i\omega t)\ ]\ dV$$

$$= -\ \rho g\ K\ A \cos\ \beta\ \exp\ (-\ Kh - i\omega t) \int \exp\ (iK\xi \cos\ \beta)\ S(\xi)\ d\xi \qquad [19]$$

where $S(\xi)$ is the cross-sectional area of the body. The volume integral is over the interior of the body, and the line integral is over the body length.

Similarly, for the surface integral in Equation [8],

$$\iint p\ [\zeta \cos (n,\xi) - \xi \cos (n,\zeta)]\ dS$$

$$= \iiint \left(\zeta \frac{\partial p}{\partial \xi} - \xi \frac{\partial p}{\partial \zeta} \right) dV$$

$$= \iiint [\rho \ddot{\theta} \xi^2 - 2\ i\omega^2 \rho\ A\xi \exp (-\ Kh + iK\xi \cos \beta - i\omega t) + \rho g\ \xi]\ dV \qquad [20]$$

$$= \int S(\xi)\ [\rho \ddot{\theta} \xi^2 - 2\ i\omega^2 \rho\ A\xi \exp (-\ Kh + iK\xi \cos \beta - i\omega t) + \rho g\ \xi]\ d\xi$$

Substituting Equations [19] and [20] in the motion Equations [7] and [8], we obtain

$$M\ddot{\xi}_o + (T/c)\ \xi_o - (T\ell/c)\theta = \rho g\ KA \cos \beta \exp (-Kh - i\omega t) \int S(\xi) \exp (iK\xi \cos \beta)\ d\xi \qquad [21]$$

$$\ddot{\theta}(1 + \rho \int S(\xi)\xi^2\ d\xi) + T\ell\ (1 + \ell/c)\ \theta - (T\ell/c)\ \xi_o - Mg\ \zeta_{CG}\ \theta = 2\ i\rho g\ KA \exp (-\ Kh - i\omega t) \int S(\xi)\ \xi \exp (iK\xi \cos \beta)\ d\xi \qquad [22]$$

where in Equation [22] we have used the equilibrium condition Equation [1] and the fact that

$$\int \xi\ S(\xi)\ d\xi = \forall\ \xi_{CB}$$

to cancel the nonoscillatory terms.

Equations [21] and [22] are coupled equations of motion for the surge and pitch motions of the body. Two simplifications can be made. First, if the body length is small compared to the wavelength, the exponential

$$e^{ik\xi \cos \beta} \cong 1$$

and it follows that

$$M \ddot{\xi}_o + (T/c)\xi_o - (T\ell/c)\theta = \rho g\ KA \forall \cos\beta \exp(-Kh - i\omega t) \qquad [23]$$

$$\ddot{\theta}(I + \rho\int S(\xi)\xi^2\, d\xi) + T\ell\ (1 + \ell/c)\theta - (T\ell/c)\ \xi_o - Mg\ \zeta_{CG}\ \theta = 2\ i\rho g\ KA \forall \xi_{CB} \exp(-Kh - i\omega t) \qquad [24]$$

In this form the only integral involved is the geometric quantity

$$I' = \rho \int S(\xi)\xi^2\, d\xi \qquad [25]$$

(i.e. the added moment of inertia), and computations are facilitated. In particular, solving the pair of coupled Equations [23] and [24] gives the pitch amplitude

$$\theta = \frac{[(T\ell/c)\cos\beta + 2i\ \xi_{CB}\ (-\omega^2 M + T/c)]\ \rho g KA \forall \exp(-Kh - i\omega t)}{(-\omega^2 M + T/c)\ [T\ell\ (1 + \ell/c) - Mg\ \zeta_{CG} - \omega^2\ (I + I')] - (T\ell/c)^2} \qquad [26]$$

As a second simplification we may assume that the cable length c is very large compared to the arm length ℓ. Then the equations of motion are uncoupled, and

$$\theta = \frac{2i\ \xi_{CB}\ \rho g\ KA \forall \exp(-Kh - i\omega t)}{T\ell - Mg\ \zeta_{CG} - \omega^2\ (I + I')} \qquad [27]$$

We note that in this circumstance the pitch response is independent of the heading angle β.

Finally we recall that the cable tension T is given by Equation [2]. Thus

$$\theta = \frac{2i\ \xi_{CB}\ \rho g\ KA \forall \exp(-Kh - i\omega t)}{\rho g \forall\ \ell - Mg\ (\ell + \zeta_{CG}) - \omega^2\ (I + I')}$$

DESCRIPTION OF MODEL AND EXPERIMENTAL TEST SET-UP

A 1/4-scale model of the MARK 56 mine was constructed, except for the tail fins, according to drawings provided by the Naval Ordnance Laboratory. The model was designed to accommodate an aircraft-type vertical gyro to measure pitch angle. Photographs of the model are shown in Figures 2 and 3.

The model was ballasted to the corresponding full-scale weight-in-air, and the center of gravity was adjusted so that the main axis of the model was horizontal while moored beneath the water surface. The inertia of the model in pitch about the center of gravity was determined by oscillating the model in air, using bifilar suspension. The center of gravity was located by using a knife edge and by freely floating the model in water and noting the angle of inclination. Model characteristics such as the inertia, weight, and location of the center of gravity are presented in Figure 4.

Dynamic characteristics of the model were altered for some tests to provide for a more thorough corroboration of theoretical computation with experimental results. This was accomplished by lengthening the mooring arm 20 inches (7 feet full scale). Characteristics of the altered model are presented in Figure 5.

In addition to pitch angle, measurements were made of the wave height and heading angle of the buoy relative to the wave direction. The wave height was measured with a resistance type probe. Heading angle was measured by manually rotating the shaft of a calibrated potentiometer to align with the axis of the model.

The model was moored with soft stainless steel wire 0.031 inches in diameter. Cables from the gyro were looped to minimize interference with the model motions. A sketch of the test set-up is shown in Figure 6.

TEST RESULTS

Tests were conducted in waves on the unaltered model at mooring depths of 5 and 10 feet below the undisturbed free surface (20 and 40 feet full scale). Water depth in the test basin was 20 feet (80 feet full scale). The model was tested through a range of wave heights and for wavelengths

of approximately 40 feet and 55 feet. These wavelengths are very close to the maximum that can be generated in the facility. The results of these tests are tabulated in Table 1.

Free oscillation of the model in pitch indicated that the resonant period of the model (4.45 seconds at 5 feet depth and 4.3 seconds at 10 feet depth) was much larger than the wave periods that could be generated in the facility. Consequently, to explore model behavior at a resonant condition, the dynamic characteristics of the model were altered as indicated previously. Tests were conducted on the altered model at 5 feet below the undisturbed free surface in a variance of wavelengths spanning the new resonant period in pitch (2.26 seconds). The results of these tests are presented in Table 2.

FULL SCALE PREDICTIONS OF MARK 56 AND 57 MINES

We shall now present a theoretical comparison of the pitch response of the MARK 56 and 57 mines. Pertinent data are as follows:

		MARK 56	MARK 57
Length	inches	89.8	101.8
Diameter	inches	23.4	20.8
Weight	pounds	975	1,040
ξ_{CG}	inches	55.2	68.0
ℓ	inches	27.6	24.1
g I	pounds square inches	**3,550,000	**5,607,000
$\forall$	cubic inches	* 33,900	* 32,600
Displacement, sea water	pounds	* 1,250	* 1,210
T, sea water	pounds	275	170
ξ_{CB}	inches	* 44.0	* 56.9
g I$'$ sea water	pounds square inches	3,030,000	4,860,000

*Bare hulls, without appendages.

**Based upon assumed radius of gyration about the center of gravity of 0.272 L, measured on the MARK 56 aluminum model.

From Equation [27] we define the following nondimensional pitch response parameter:

$$\frac{|\theta|}{KA\ e^{-Kh}} = \frac{2\xi_{CB}\ \rho g\ \forall}{T\ell - \omega^2\ (I + I')}$$

where we assume $\zeta_{CG} = 0$. This parameter is the ratio of pitch angle to the maximum wave slope at the depth h. Using the relation for the wavelength $\lambda = 2\pi g/\omega^2$, it follows that

$$\frac{|\theta|}{KA\ e^{-Kh}} = \frac{2\xi_{CG}\ \lambda\rho g\ \forall}{T\ell\ \lambda - 2\pi g\ (I + I')}$$

or, from the above data, and with λ in feet,

$$\frac{|\theta|}{KA\ e^{-Kh}} = \frac{14.5\ \lambda}{|454 - \lambda|} \qquad \text{[MARK 56]}$$

$$= \frac{33.6\ \lambda}{|1340 - \lambda|} \qquad \text{[MARK 57]}$$

A dimensional measure of pitch response is

$$\frac{|\theta|}{Ae^{-Kh}} = \frac{4\pi\ \xi_{CB}\ \rho g\ \forall}{T\ell\ \lambda - 2\pi\ g\ (I + I')} \quad \text{radians/unit length}$$

$$= \frac{720\ \rho g\ \forall\ \xi_{CB}}{T\ell\lambda - 2\pi g\ (I + I')} \quad \text{degrees/unit length}$$

where Ae^{-Kh} is the effective wave height at the depth h. Thus for the two mines

$$\frac{|\theta|}{Ae^{-Kh}} = \frac{5200}{|454 - \lambda|}\ \text{degrees/foot} \qquad \text{[MARK 56]}$$

$$= \frac{12{,}100}{|1340 - \lambda|}\ \text{degrees/foot} \qquad \text{[MARK 57]}$$

These two response functions are plotted in Figure 7, along with values of the exponential function e^{-Kh} for various depths. Also shown by dashed lines are approximate expected values for the response near resonance, as inferred from the experiments with the altered (long mooring arm) model. Figure 8 shows the comparison of theoretical* and experimental data for the MARK 56 mine and for the altered (long mooring arm) version of this mine. The agreement is generally good except for excessive scatter of the experimental data at the longest wavelength, which is attributed to the limitations of the wavemaker and for the breakdown of the theory in the resonance domain of the altered model.

PREDICTED PITCHING MOTION IN A SEAWAY

Predictions have been made of the pitching motions of both mines in several idealized sea conditions and at several mooring depths by applying the principles of linear superposition. Using this procedure, the spectrum of pitching motion is obtained by multiplying the wave-height spectrum by the square of the amplitude response in pitch. The form of the wave spectrum used in these calculations is that proposed by Pierson and Moskowitz[1] for a fully developed wind generated sea and is given by

$$S(\omega) = \frac{8.10 \times 10^{-3}\, g^2}{\omega^5} e^{-0.74\, (\omega_0/\omega)^4}$$

where g is the acceleration due to gravity; $\omega_o = g/V$, where V is the wind velocity 19.5 meters above the sea surface.

The calculations were made for Sea States 4 through 7, corresponding to significant wave heights of 6, 10, 15, and 30 feet, respectively,

---

*These theoretical predictions differ from the above formulae insofar as the fluid density of fresh water is used; the only significant effect is on the cable tension. The theoretical prediction for the long arm is based upon the coupled equations of motion, the pitch amplitude being given by Equation [26]; in this instance, the ratio of ℓ/c was sufficiently large (0.157) to affect the motions.

and at mooring depths of 50, 100, 150 and 200 feet below the surface. The results are summarized in Table 3 which presents the root-mean-square values of pitching motion for the various conditions. Values greater than 25 degrees are not reported since this implies pitching motion in excess of 90 degrees which is definitely not within the limits of the linear theory used in these calculations.

The data in Table 3 show that the pitch motion of the MARK 56 buoy is greater than the MARK 57 in the lower sea conditions. The resonant pitch period of the MARK 56 buoy is much shorter than the MARK 57 and, consequently, it responds more to the higher frequency-wave content in the seaway. As the sea state increases, the energy at the lower wave frequencies increases very rapidly, and the MARK 57 buoy with its longer resonant period responds with greater pitching motion than the MARK 56. Increasing the mooring depth attenuates the motion for both buoys since the wave energy is attenuated with depth.

PROBABILITY OF EXCEEDING A CERTAIN PITCH AMPLITUDE (PEAK VALUE)

The pitch spectra have sufficiently narrow bands that we may assume that the envelope, hence the peak values of the pitching motion, follow a "Rayleigh distribution."
The Rayleigh distribution is given by

$$P\ (\theta) = \frac{\theta}{\sigma^2_\theta}\ e^{-\ \theta^2/2\sigma^2_\theta}$$

where θ is the envelope or a peak value of the pitch motion and σ^2_θ is the mean-square value of pitch and may be obtained by squaring the values in Table 3.

The probability that the envelope of the pitching motion exceeds a value θ_o is obtained by integrating the expression for the Rayleigh distribution from θ_o to ∞, which yields

$$P\ (\theta > \theta_o) = e^{-\ \theta^2_o/2\sigma^2_\theta}$$

The theoretical probability distribution would always indicate that a certain value of pitch amplitude (peak value) would be exceeded, even though this probability may be extremely small. In actuality the distribution will be truncated at some finite value of pitch amplitude. This cut-off point is not known with any certainty; however, it is reasonable to assign some low value of probability, for instance, 0.0001, beyond which no value of pitch motion is considered to exist.

The probability of exceeding pitch amplitudes of 51 and 60 degrees for various sea states and buoy depths is presented in Tables 4 and 5. These results show that neither body will exceed these angles in Sea State 4. As the sea state increases these angles will be exceeded first at the shallow depths and, finally, in Sea State 7 at all depths for which the calculations were made.

CONCLUSIONS

1. The theoretical predictions of the pitch motions of the MARK 56 and MARK 57 mines are reliable, except near resonance, which will occur at wavelengths of approximately 450 and 1340 feet, respectively. At resonance the motions are governed by viscous damping which can not be predicted with confidence from theory or experiments.

2. For wavelengths less than 700 feet the MARK 57 mine is superior with regard to minimization of pitching motions. But for larger wavelengths the MARK 56 is superior and with a larger difference involved.

3. In a seaway both the relative and absolute performance of the two mines depends on the sea state and the depth of submergence, the MARK 56 being better in situations involving higher sea states and deeper submergence depths.

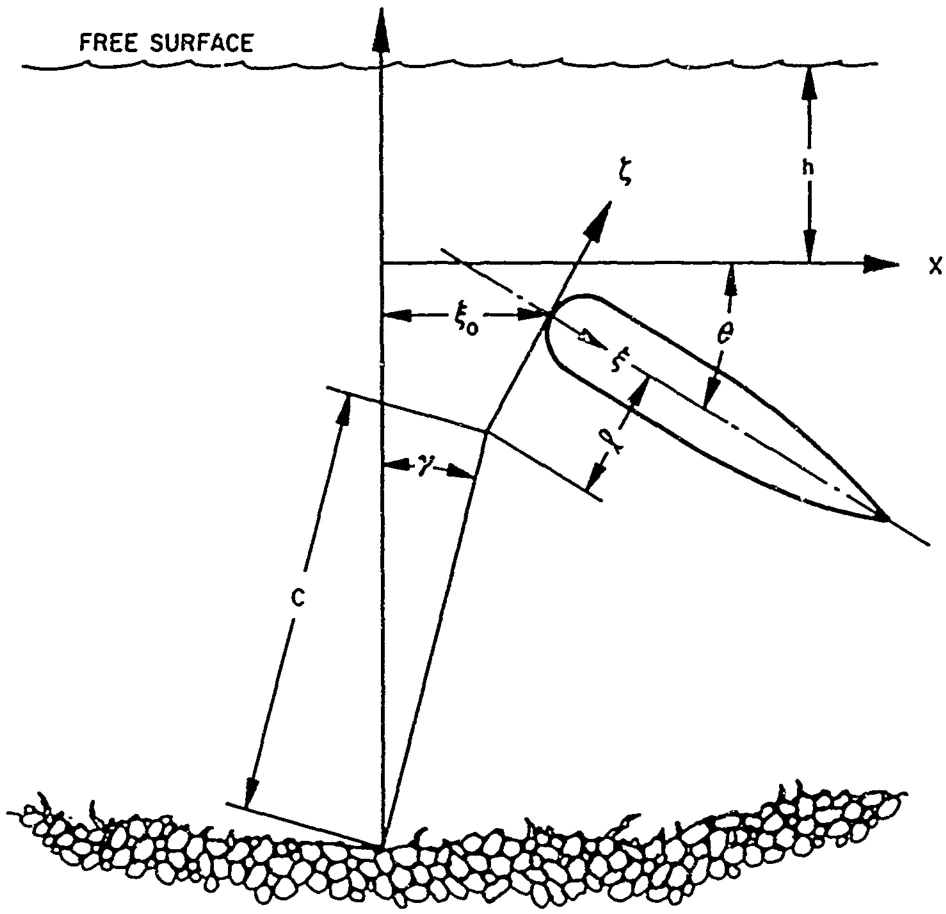

Figure 1 - Coordinate System and Geometric Set-up

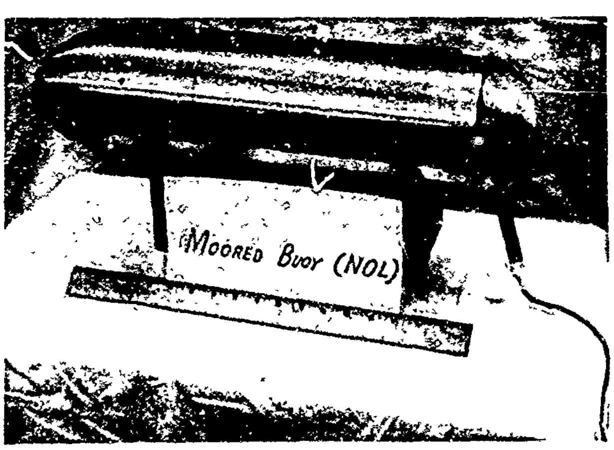

Figure 2 - Model of MARK 56 Mine

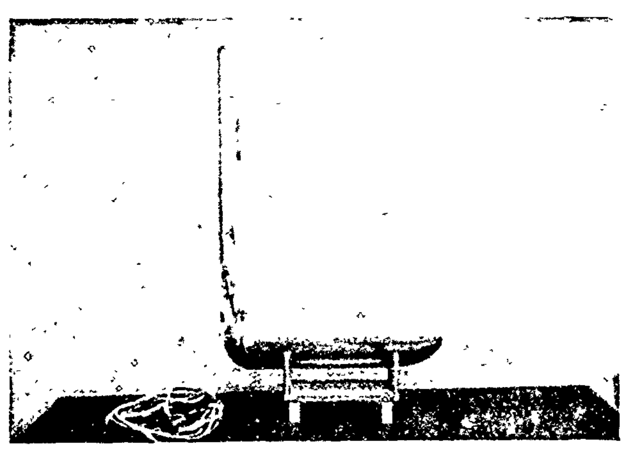

Figure 3 - Model of Altered MARK 56 Mine

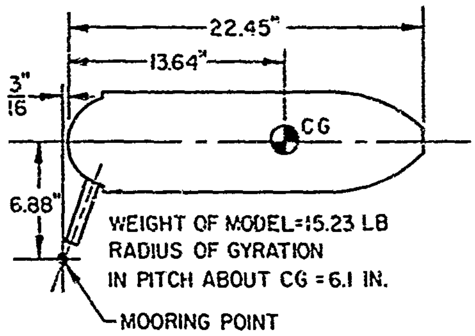

Figure 4 - Characteristics of Model

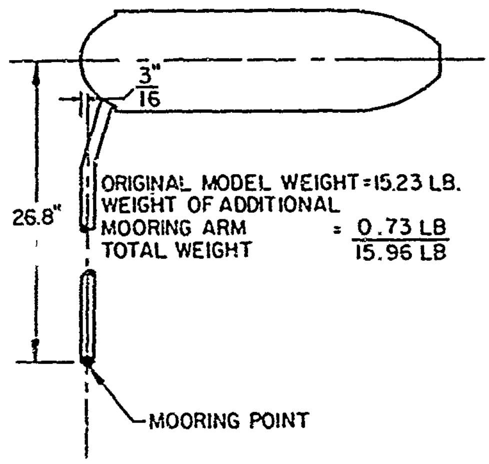

Figure 5 - Characteristics of Altered Model

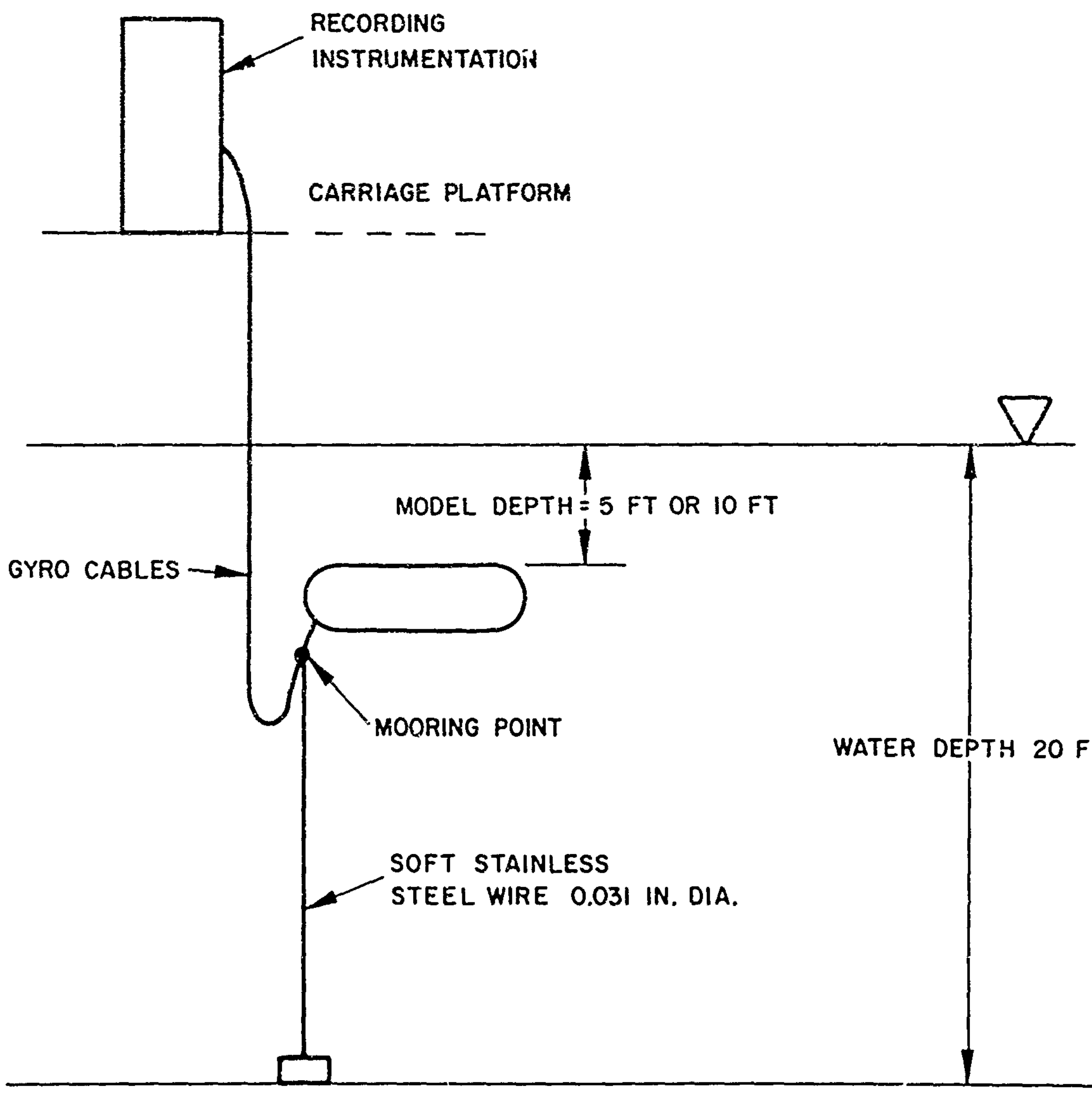

Figure 6 - Sketch of Test Set-Up

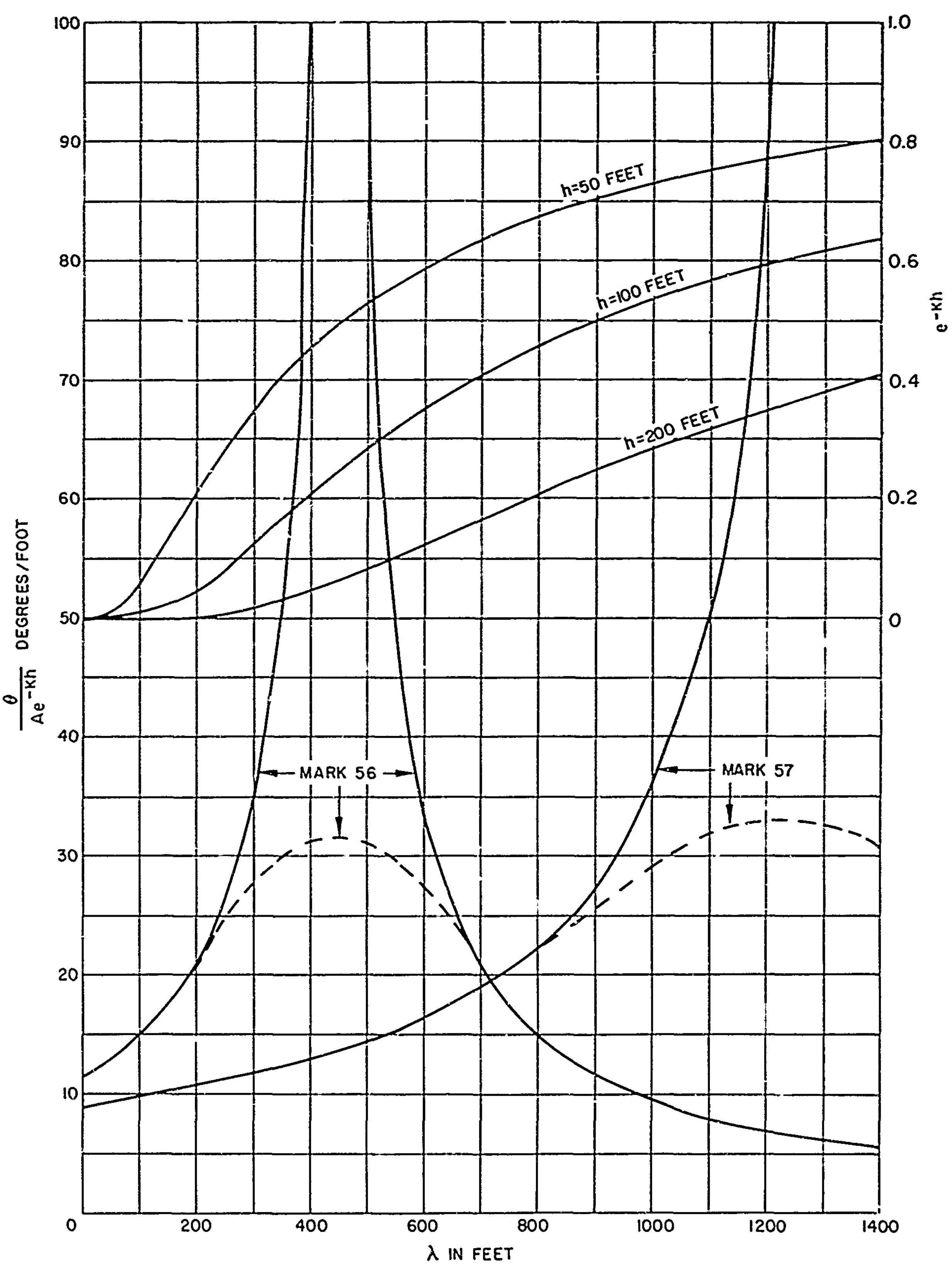

Figure 7 - Predicted Pitch Response of Full-Scale Mines

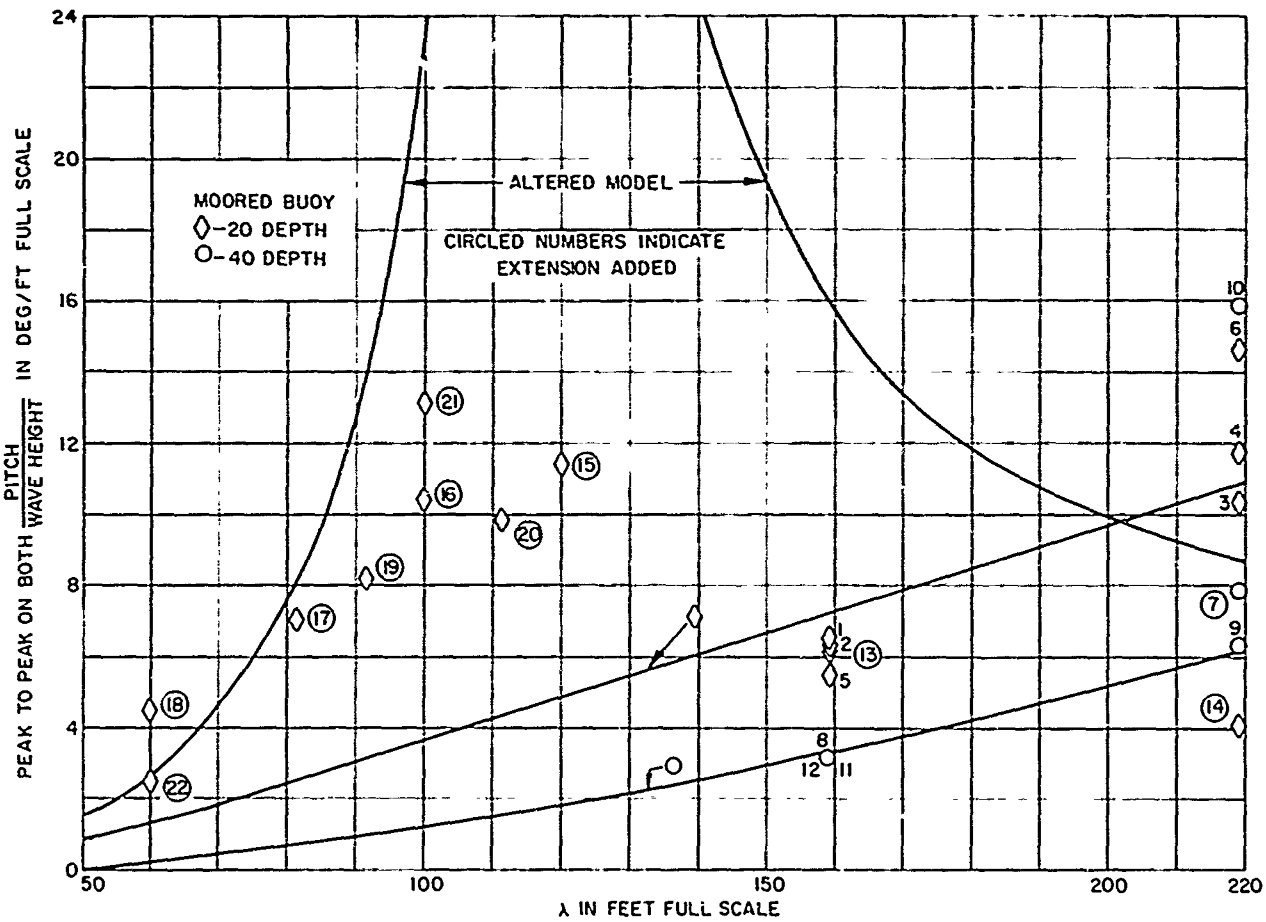

Figure 8 - Comparison of Measured and Predicted Pitch Response for Model 56 Mine at 20-Foot and 40-Foot Depths and of Altered (Long Mooring Arm) Mine at 20-Foot Depth

Run numbers denote experimental measurements and solid lines denote theoretical predictions.

TABLE 1

Test Results (Unaltered Model)

Depth of model beneath free surface { 5 feet, model scale; 20 feet, full scale }

Depth of water { 20 feet, model scale; 80 feet, full scale }

Period of free oscillation in pitch { 4.4 seconds, model scale; 8.8 seconds, full scale }

Wave length		Wave height*		Heading angle, ** degrees	Pitch amplitude, degrees
Model, feet	Full scale, feet	Model, inches	Full scale feet		
39.8	159	7.2	2.4	3	8.6
39.8	159	15.0	5.0	2	18.4
39.8	159	25.1	8.4	1	29.1
54.7	219	6.4	2.1	0	11.1
54.7	219	12.7	4.2	7	24.9
54.7	219	18.3	6.1	***0	44.7

Depth of model beneath free surface { 10 feet, model scale; 40 feet, full scale }

Period of free oscillation in pitch { 4.3 seconds, model scale; 8.6 seconds, full scale }

Wave length		Wave height*		Heading angles,** degrees	Pitch amplitude, degrees
Model, feet	Full scale, feet	Model, inches	Full scale feet		
39.8	159	7.9	2.6	6	4.0
39.8	159	14.8	4.9	0	7.6
39.8	159	23.7	7.9	0	12.8
54.7	219	5.0	1.7	20	6.6
54.7	219	13.5	4.5	6	14.1
54.7	219	19.0	6.3	0	24.9

*Double amplitude.

**Heading angle relative to wave direction.

***Head seas.

TABLE 2

Test Results (Altered Model)

Depth of model beneath free surface: 5 feet, model scale; 20 feet, full scale

Depth of water: 20 feet, model scale; 80 feet, full scale

Period of free oscillations in pitch: 2.3 seconds, model scale; 4.6 seconds, full scale

Wave length		Wave height*		Heading angle,** degrees	Pitch amplitude, degrees
Model, feet	Full scale, feet	Model, inches	Full scale, feet		
39.8	159	21.1	7.0	22	22.0
54.7	219	20.1	6.7	13	13.5
30.0	120	7.7	2.6	69	14.8
25.0	100	8.3	2.8	9	14.6
20.0	80	9.0	3.0	***0	10.5
22.6	90.4	9.3	3.1	4	12.7
27.6	110	9.5	3.2	7	15.5
25.0	100	4.8	1.6	2	10.4
15.0	60	9.0	3.0	35	3.8

*Double amplitude.

**Heading angle relative to wave direction.

***Head Seas.

TABLE 3

Root-Mean-Square Pitching Motion in Degrees

Mine / Sea State	50 feet		100 feet		150 feet		200 feet	
	MARK 56	MARK 57	MARK 56	MARK 57	MARK 56	MARK 57	MARK 56	MARK 57
4	9.1	4.2	3.5	1.6	1.5	0.7	0.7	0.3
5	>25	14	13	7.2	6.8	4.2	2.5	2.5
6	>25	>25	25	22	14	14	8.0	9.5
7	>25	>25	>25	>25	>25	>25	19	>25

TABLE 4

Probability (P) That the Pitch Amplitude* θ Exceeds a Value of θ_o = 51 Degrees

Mine / Sea State	50 feet		100 feet		150 feet		200 feet	
	MARK 56	MARK 57	MARK 56	MARK 57	MARK 56	MARK 57	MARK 56	MARK 57
4	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001
5	>0.1250	0.0013	0.0046	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001
6	>0.1250	>0.1250	0.1250	0.068	0.0013	0.0013	<0.0001	<0.0001
7	>0.1250	>0.1250	>0.1250	>0.1250	>0.1250	>0.1250	0.0274	>0.1250

*Envelope or peak values of the pitch signal.

TABLE 5

Probability (P) That the Pitch Amplitude* θ Exceeds a Value of θ_o = 60 Degrees

Mine / Sea State	50 feet		100 feet		150 feet		200 feet	
	MARK 56	MARK 57	MARK 56	MARK 57	MARK 56	MARK 57	MARK 56	MARK 57
4	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001
5	>0.0562	0.0001	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001
6	>0.0562	>0.056	0.0562	0.0242	0.0001	0.0001	<0.0001	<0.0001
7	>0.0562	>0.056	>0.0562	>0.0562	>0.0562	>0.0562	0.0068	>0.0562

*Envelope or peak values of the pitch signal.

REFERENCE

1. Pierson, W. J. and Moskowitz, L., "A Proposed Spectral Form for Fully Developed Wind Seas Based on Similarity Theory of S. A. Kitaigorodskii," Journal of Geophysical Research, Vol. 69, No. 24, (1964) pages 5181-5190.

Unclassified
Security Classification

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1 ORIGINATING ACTIVITY *(Corporate author)*
David Taylor Model Basin
Washington D.C. 20007

2a. REPORT SECURITY CLASSIFICATION
Unclassified

2b GROUP

3. REPORT TITLE
PITCHING MOTIONS OF A MOORED SUBMERGED MINE IN WAVES

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*
Final Report

5. AUTHOR(S) *(Last name, first name, initial)*
J. N. Newman; P. Plaia; E. E. Zarnick

6. REPORT DATE
March 1966

7a. TOTAL NO. OF PAGES
29

7b. NO. OF REFS
1

8a. CONTRACT OR GRANT NO.
NOL WR 5-0044

b. PROJECT NO.

c.

d.

9a. ORIGINATOR'S REPORT NUMBER(S)
2151

9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*

10. AVAILABILITY/LIMITATION NOTICES
Distribution of this document is unlimited.

11. SUPPLEMENTARY NOTES

12. SPONSORING MILITARY ACTIVITY
U.S. Naval Ordnance Laboratory,
White Oak

13. ABSTRACT

This report describes a theoretical and experimental investigation of the pitching motions of a moored, submerged mine. The theoretical predictions are based upon linearized-wave theory as well as the assumptions that the body is slender and axisymmetric and is ballasted to be at equilibrium in the horizontal plane. The mooring cable is assumed to be massless and inelastic; the fluid is assumed to be inviscid. The theory results in an equation of undamped motion. Parallel experimental results were obtained on a 2-foot long model in wavelengths ranging from 15 to 55 feet, and these results confirm the theoretical predictions except in the vicinity of resonance, where viscous damping is important. Full-scale predictions are made for the root-mean-square pitching motions in Sea States 4 through 7 for two proposed mine configurations at various depths of submergence. The predicted values are from 1 to 0 degrees in Sea State 4, depending on depth and mine configuration, increasing to greater than 25 degrees in Sea State 7.

DD FORM 1 JAN 64 1473

Unclassified
Security Classification

Unclassified

Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Mines						
Slender Body-Theory						
Moored Mines in Waves						
Submerged Mines in Waves						
Pitch Motions of Submerged Body						

INSTRUCTIONS

1. ORIGINATING ACTIVITY. Enter the name and address of the contractor, subcontractor, grantee, Department of Defense activity or other organization *(corporate author)* issuing the report.

2*a*. REPORT SECURITY CLASSIFICATION: Enter the overall security classification of the report. Indicate whether "Restricted Data" is included. Marking is to be in accordance with appropriate security regulations.

2*b*. GROUP: Automatic downgrading is specified in DoD Directive 5200.10 and Armed Forces Industrial Manual. Enter the group number. Also, when applicable, show that optional markings have been used for Group 3 and Group 4 as authorized.

3. REPORT TITLE: Enter the complete report title in all capital letters. Titles in all cases should be unclassified. If a meaningful title cannot be selected without classification, show title classification in all capitals in parenthesis immediately following the title.

4. DESCRIPTIVE NOTES: If appropriate, enter the type of report, e.g., interim, progress, summary, annual, or final. Give the inclusive dates when a specific reporting period is covered.

5. AUTHOR(S): Enter the name(s) of author(s) as shown on or in the report. Enter last name, first name, middle initial. If military, show rank and branch of service. The name of the principal author is an absolute minimum requirement.

6. REPORT DATE: Enter the date of the report as day, month, year; or month, year. If more than one date appears on the report, use date of publication.

7*a*. TOTAL NUMBER OF PAGES: The total page count should follow normal pagination procedures, i.e., enter the number of pages containing information.

7*b*. NUMBER OF REFERENCES: Enter the total number of references cited in the report.

8*a*. CONTRACT OR GRANT NUMBER. If appropriate, enter the applicable number of the contract or grant under which the report was written.

8*b*, 8*c*, & 8*d*. PROJECT NUMBER. Enter the appropriate military department identification, such as project number, subproject number, system numbers, task number, etc.

9*a*. ORIGINATOR'S REPORT NUMBER(S): Enter the official report number by which the document will be identified and controlled by the originating activity. This number must be unique to this report.

9*b*. OTHER REPORT NUMBER(S): If the report has been assigned any other report numbers *(either by the originator or by the sponsor)*, also enter this number(s).

10. AVAILABILITY/LIMITATION NOTICES: Enter any limitations on further dissemination of the report, other than those imposed by security classification, using standard statements such as:

(1) "Qualified requesters may obtain copies of this report from DDC."

(2) "Foreign announcement and dissemination of this report by DDC is not authorized."

(3) "U. S. Government agencies may obtain copies of this report directly from DDC. Other qualified DDC users shall request through ________________."

(4) "U. S. military agencies may obtain copies of this report directly from DDC. Other qualified users shall request through ________________."

(5) "All distribution of this report is controlled. Qualified DDC users shall request through ________________."

If the report has been furnished to the Office of Technical Services, Department of Commerce, for sale to the public, indicate this fact and enter the price, if known.

11. SUPPLEMENTARY NOTES: Use for additional explanatory notes.

12. SPONSORING MILITARY ACTIVITY. Enter the name of the departmental project office or laboratory sponsoring *(paying for)* the research and development. Include address.

13 ABSTRACT: Enter an abstract giving a brief and factual summary of the document indicative of the report, even though it may also appear elsewhere in the body of the technical report. If additional space is required, a continuation sheet shall be attached.

It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the military security classification of the information in the paragraph, represented as *(TS)*, *(S)*, *(C)*, or *(U)*.

There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.

14 KEY WORDS: Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical context. The assignment of links, roles, and weights is optional.

Unclassified

Security Classification